THE VIII INTERNATIONAL CONFERENCE OF MATHEMATICS AND COMPUTER SCIENCE

„CONGRESSIO-MATHEMATICA"

Olsztyn, Poland 19 – 25.09.2022

Department of Complex Analysis
Faculty of Mathematics and Computer Sciences
University of Warmia and Mazury in Olsztyn
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Faculty of Mathematics and Computer Sciences
University of Warmia and Mazury in Olsztyn

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Faculty of Technical Physics, Information Technology and Applied Mathematics
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CONFERENCE PROGRAM

Monday, September 19th, 2022

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13:00 – 17:00 Lunch

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Plenary lectures (Hotel HP Park)

Chairman: ***Maria Nowak***

17:00 – 17:40 **David Shoikhet**: *Holomorphic mappings and semigroups, rigidity and fixed points*

17:50 – 18:30 **Feliks Przytycki**: *On Hausdorff dimension of Julia sets*

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20:00 – 00:00 Welcome dinner

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Tuesday, September 20th, 2022

Plenary lectures (Hotel HP Park)

Chairman: ***Feliks Przytycki***

09:00 – 09:40 **Maria Nowak**: *Harmonically weighted Dirichlet space associated with finitely atomic measures and de Branges-Rovnyak spaces*

09:50 – 10:30 **Miodrag Mateljević**: *Boundary behaviour of partial derivatives for solutions to certain Laplacian-gradient inequalities and spatial QC maps*

Chairman: ***Mark Elin***

10:40 – 11:20 **Michael Dorff**: *Zeros of a one-parameter family of harmonic trinomials*

Chairman: ***David Shoikhet***

11:30 – 12:10 **Józef Zając**: *Technical applications of some results on plane harmonic mappings*

12:20 – 13:00 **Adam Lecko, Dariusz Partyka**: *Generalized Fekete-Szegö functionals*

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13:00 – 14:00 Lunch

***

Section I (Hotel HP Park)

Chairman: ***Miodrag Mateljević***

14:15 –14:40 **Marek Svetlik**: *The Schwarz lemma for HQR mappings*

14:45 – 15:10 **Milijan Knežević**: *Some properties of the quasi-conformal diffeomorphisms of the unit disc*

15:15 – 15:40 **Adel Khalfallah**: *Schwarz-Pick lemma for harmonic and hyperbolic harmonic functions*

15:45 – 16:10 **Nikola Mutavdžić**: *Note on some classes of holomorphic functions related to Jack's and Schwarz's lemma*

Section II (Hotel HP Park)

Chairman: ***Jacek Chudziak***

16:20 – 16:45 **Tania Osipchuk**: *Some topological properties of generalized closed-to-convex sets*

16:50 – 17:15 **Ivan Matychyn, Viktoriia Onyshchenko**: *Nonstationary fractional-order systems*

17:20 – 17:45 **Anna Muranova**: *Spectrum of discrete Laplacian over an ordered field*

Section III (Hotel HP Park)

Chairman: ***Dariusz Partyka***

17:55 – 18:20 **Barbara Śmiarowska**: *Coefficient functionals for alpha-convex functions associated with the exponential function*

18:25 – 18:50 **Andrzej Michalski**: *Some estimates for coefficients of bounded analytic functions*

Section – History and didactics of mathematics I (Hotel HP Park) (organizers: Renata Długosz and Stanisław Domoradzki)

14:15 –14:55 **Izabela Jóźwik**: *100 lat twierdzenia Banacha o punkcie stałym*. Part I
Małgorzata Terepeta, *100 lat twierdzenia Banacha o punkcie stałym*. Part II

15:00 – 15:20 **Monika Lindner, Renata Długosz**: *Zajęcia z arytmetyki finansowej w kontekście zmieniającej się sytuacji ekonomicznej Polski*

15:25 – 15:50 **Renata Długosz, Monika Lindner, Piotr Ostrowski**: *Consequences of moving away from stationary math teaching in high schools during pandemic*

15:55 – 16:20 **Anna Szpila:** *Assessment of learning outcomes after the first semester of studies in mathematics conducted by the University of Rzeszów*

ONLINE MODE:

16:30 – 16:55 **Stanisław Domoradzki**: *The importance of the Commission of National Education for the development of mathematics in Poland in the years 1773-1794*

17:00 – 17:25 **Artur Wachowicz**: *O pewnym podstawowym problemie w początkowym etapie nauczania analizy matematycznej*

17:30 – 18:10 **Martina Bečvářová, Jindřich Bečvář**: *Vojtěch Jarník and his studies in Göttingen*

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20:30 – 00:00 Banquet

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Wednesday, September 21st, 2022

Plenary lectures (Hotel HP Park)

Chairman: ***Michael Dorff***

09:00 – 09:40 **Walter Bergweiler**: *Real zeros of solutions of linear differential equations*

09:50 – 10:30 **Jouni Rättyä**: *Bergman projection and BMO in hyperbolic metric*

Chairman: ***Walter Bergweiler***

10:40 – 11:20 **Marek Golasiński**: *The exceptional Lie group F_4: its applications and generalizations*

11:30 – 12:10 **Renata Długosz, Piotr Liczberski**: *Fekete-Szegö problem for Bavrin's holomorphic functions and biholomorphic mappings in C^n*

Chairman: ***Marek Golasiński***

12:20 – 13:00 **Zbigniew Suraj**: *On some approach to approximate real-time decision making: theory and implementation*

***

13:00 – 14:00 Lunch

***

Section IV (FoMaCS / hall C1)

Chairman: ***Teodor Bulboacă***

14:15 – 14:40 **Suman Das**: *Riesz-Fejér type inequalities for harmonic mappings*

14:45 – 15:10 **Sunanda Naik**: *A study of boundedness property of integral type operators on analytic function spaces*

15:15 – 15:40 **Lateef Ahmad Wani**: *Applications of hypergeometric functions in the theory of differential subordinations*

Section V (FoMaCS / hall C1)

Chairman: ***Szymon Głąb***

15:50 – 16:15 **Jacek Chudziak**: *On differentiable solutions of the translation equation*

16:20 – 16:45 **Jacek Marchwicki**: *Achievement sets of reciprocals of a complete sequences*

16:50 – 17:15 **Małgorzata Terepeta**: *Świątkowski-type conditions*

Section VI (FoMaCS / hall C1)

Chairman: ***Ivan Matychyn***

17:25 – 17:50 **Aleksandra Huczek**: *The Wolff-Denjoy type theorem for semigroups in geodesic spaces*

17:55 – 18:20 **Joanna Markowicz**: *On some geometric properties of interpolation spaces obtained with the general discrete K-method*

Section VII (FoMaCS / hall C1)

Chairman: ***Piotr Liczberski***

18:30 – 18:55 **Renata Długosz, Bartosz Bartoszek**: *On a multiplicative distribution of functions in complex plane*

19:00 – 19:25 **Katarzyna Trąbka-Więcław, Paweł Zaprawa**: *On difference of successive coefficients in M(a) and N(a)*

Section – History and didactics of mathematics II (FoMaCS / hall C2)
(organizers: Renata Długosz and Stanisław Domoradzki)

14:30 –14:55 **Karolina Mroczyńska:** *Reasoning and argumentation of a student with Asperger Syndrome in the mathematics lessons*

ONLINE MODE:

15:00 – 15:25 **Katarzyna Dems-Rudnicka, Izabela Jóźwik**: *Królowa Nauk na co dzień - o cyklu wykładów popularyzujących matematykę wśród młodzieży*

15:30 – 16:10 **Marek Małolepszy**: *Rola kontekstu w kształceniu matematycznym na studiach technicznych*

16:15 – 16:55 **Wiesław Wójcik**: *Work on the theory of probability at the Polish School of Mathematics*

17:00 – 17:25 **Violetta Lipińska**: *The form of a competence exam at the Master's degree studies at the Lodz University of Technology*

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20:00 – 00:00 Barbecue

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Thursday, September 22nd, 2022

Plenary lectures (FoMaCS / hall C1)

Chairman: ***Jouni Rättyä***

09:00 – 09:40 **Derek K. Thomas**: *Coefficient invariances for convex functions*

09:50 – 10:30 **Mark Elin**: *Filtration of generators and an inverse Fekete-Szegö problem*

Chairman: ***Derek K. Thomas***

10:40 – 11:20 **Teodor Bulboacă**: *Subordination properties and initial coefficients bounds for a subclass of convex functions*

11:30 – 12:10 **Szymon Głąb:** *On strong algebrability of families of non-measurable functions of two variables*

12:15 – 12:55 ***Poster session*** (FoMaCS / main hall)

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13:00 – 14:00 Lunch

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Saturday, September 24th, 2022

Plenary lectures

Chairman: ***Andrzej Wiśnicki***

09:00 – 09:40 **Filippo Bracci**: *Abstract boundaries and applications*

09:50 – 10:15 **Marek Balcerzak**: *Cutting sets of continuous functions on the unit interval*

Chairman: ***Marek Balcerzak***

10:25 – 11:05 **Andrzej Wiśnicki**: *A fixed point theorem in B(H,l_infty)*

Chairman: ***Filippo Bracci***

11:15 – 11:55 **Abdallah Lyzzaik**: *Moduli of doubly connected domains under univalent harmonic maps*

12:05 – 12:45 **Sairam Kaliraj**: *Growth of harmonic functions and its applications*

Chairman: ***Sairam Kaliraj***

12:55 – 13:35 **Saminathan Ponnusamy**: *Landau-Bloch Theorems for Harmonic Mappings*

13:45 – 14:25 **Toshiyuki Sugawa**: *Harmonic spirallike functions and harmonic strongly starlike functions*

Chairman: ***Saminathan Ponnusamy***

14:35 – 15:15 **Galina Filipuk**: *On the Painleve XXV - Ermakov equation*

Section – Computer Science
(organizer: Piotr Artiemjew)

15:25 – 16:05 **Saima Mustafa:** *Recent techniques of fuzzy set theory and its applications*

16:10 – 16:35 **Adam Ligęza**: *Hamiltonians of Painlevé V equation*

16:40 – 17:05 **Krzysztof Pancerz**, **Jaromir Sarzyński**: *Rough set flow graph visualizer in classification and prediction software system (CLAPSS)*

17:10 – 17:35 **Mikhail Kolev**, **Irina Naskinowa**: *On some computational and mathematical applications*

17:40 – 18:05 **Bożena Staruch**: *On the problem of effective use of fabric remnants in the production of upholstered furniture*

Sunday, September 25th, 2022

Plenary lectures

Chairman: ***See Keong Lee***

09:00 – 09:40 **Young Jae Sim, Derek K. Thomas**: *Recent bounds for Hankel determinants for starlike functions with respect to symmetrical points*

09:50 – 10:30 **Vasudevarao Allu, Himadri Halder**: *Bohr phenomenon for harmonic Bloch functions on simply connected domains*

Chairman: ***Vasudevarao Allu***

10:40 – 11:20 **Bappaditya Bhowmik**: *Existence of nonzero poles produce nontrivial lower bounds for coeffcients of certain harmonic univalent functions*

11:30 – 12:10 **Anbhu Swaminathan**: *Geometric properties of ratios of hypergeometric functions*

Chairman: ***Bappaditya Bhowmik***

12:20 – 13:00 **Srikandan Sivasubramanian**: *On a subclass of close-to-convex functions involving nephroid and cardioid domains*

13:10 – 13:50 **See Keong Lee**: *The Bohr operator on analytic functions*

Section VIII

Chairman: ***Anbhu Swaminathan***

14:00 – 14:25 **Sanjeev Singh**: *The generalized Marcum function of the second kind: Monotonicity patterns and tight bounds*

14:30 – 14:55 **Mohsan Raza**: *Starlikeness associated with the reciprocal of Bernoulli functions*

15:00 – 15:25 **Virendra Kumar**: *Recent development on coefficient functionals in univalent function theory*

Section IX

Chairman: ***Srikandan Sivasubramanian***

15:35 – 16:00 **Sushil Kumar**: *Properties of certain functions associated with the right half plane its applications and generalizations*

16:05 – 16:30 **Swadesh Kumar Sahoo**: *Geometry of Cassini ovals and applications*

Section X

Chairman: ***Sushil Kumar***

16:35 – 17:00 **Molla Ahamed**: *Transcendental entire solutions of Fermat-type differential-difference equations in C^2*

17:05 – 17:30 **Kapil Jaglan**: *On the partial sums of univalent harmonic mappings*

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17:35 – 17:45 CLOSING

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ABSTRACTS

VASUDEVARAO ALLU, HIMADRI HALDER

Indian Institute of Technology Bhubaneswar (Bhubaneswar, India)

Bohr phenomenon for harmonic Bloch functions on simply connected domains

The Bohr phenomenon for analytic functions of the form $f(z) = \sum_{n=0}^{\infty} a_n z^n$, first introduced by Harald Bohr in 1914, deals with finding the largest radius r_f, $0 < r_f < 1$, such that the inequality $\sum_{n=0}^{\infty} |a_n z^n| \leq 1$ holds whenever the inequality $|f(z)| \leq 1$ holds in the unit disk $\mathbf{D} := \{z \in \mathbf{C} : |z| < 1\}$. The exact value of this largest radius known as Bohr radius, has been shown to be $r_f = 1/3$. For $\alpha \in (0, \infty)$, let $\mathcal{B}_{\mathcal{H},\Omega}(\alpha)$ denote the class of α-Bloch mappings on a proper simply connected domain $\Omega \subseteq \mathbf{C}$. In this talk, we discuss the class $\mathcal{B}^*_{\mathcal{H},\Omega}(\alpha)$ of harmonic α-Bloch-type mappings on a proper simply connected domain $\Omega \subseteq \mathbf{C}$ and study several interesting properties of the classes $\mathcal{B}_{\mathcal{H},\Omega}(\alpha)$ and $\mathcal{B}^*_{\mathcal{H},\Omega}(\alpha)$ when Ω is a proper simply connected domain and the shifted disk Ω_γ containing $\mathbf{D}$, where

$$\Omega_\gamma := \left\{ z \in \mathbf{C} : \left| z + \frac{\gamma}{1-\gamma} \right| < \frac{1}{1-\gamma} \right\}$$

and $0 \leq \gamma < 1$. We discuss the Landau's theorem for the harmonic Bloch space $\mathcal{B}_{\mathcal{H},\Omega_\gamma}(\alpha)$ on the shifted disk Ω_γ.

AMS Subject Classification: Primary 30C45, 30C50, 30C80

Key Words and Phrases: analytic, univalent, harmonic functions; starlike, convex, close-to-convex functions; coefficient estimate, growth theorem, Bohr radius.

MAREK BALCERZAK

Lodz Univerity of Technology (Lodz, Poland)

Cutting sets of continuous functions on the unit interval

We present results obtained together with P. Nowakowski and M. Popławski [1].

We say that a function $f\colon [0,1] \to \mathbf{R}$ has a *cutting point* at $x \in [0,1]$ if $f(x) = 0$ and for each neighbourhood U of x there are $y, z \in U$ such that $f(y) < 0 < f(z)$. Denote by $E(f)$ the set of all cutting points of f. Zabeti (2016) constructed a function $f \in C[0,1]$ (and also $f \in C^\infty[0,1]$) such that $E(f)$ contains a Cantor set of positive measure. We extend his investigations.

We show that, for every function $f \in C[0,1]$, the set $E(f)$ is closed and nowhere dense. Additionally, if $\operatorname{Int} f^{-1}\{0\} = \emptyset$ then isolated points of $E(f)$ are contained in $(0,1)$. Conversely, if $F \subseteq [0,1]$ is a closed nowhere dense set with isolated points contained in $(0,1)$, then there exists a function $f \in C^\infty[0,1]$ such that $E(f) = F = f^{-1}[\{0\}]$.

REFERENCES

[1] M. Balcerzak, P. Nowakowski, M. Popławski, *Cutting sets of continuous functions on the unit interval*, Indagationes Math. **33** (2022), 625–635.

Martina BEČVÁŘOVÁ, Jindřich BEČVÁŘ

Czech Technical University in Prague (Praha, Czech Republic)

Vojtěch Jarník and his studies in Göttingen

We intend to present unique archival materials, to show the role of Jarník's studies in Göttingen for his future mathematical career and collaboration with mathematicians from all over the world.

From the second half of the 19th century, the most talented mathemati¬cians from the Czech lands (later from Czechoslovakia) went abroad, thanks to government scholarships and funding. They travelled mainly to Germany, France, Italy or USA and looked there for better career, broadening the horizon of their knowledge and contacts with the best mathematical centres of Western Europe as well as possibility to publish their scientific works there.

Among the most no¬table archival documents from the interwar period, located in the Archive of the Academy of Sciences of the Czech Republic, are fourteen notebooks with notes written by Vojtěch Jarník (1897–1970, the later prominent Czech mathematician), during his studies at Göttingen in the years 1923–1925 and 1927–1928. The notebooks contain his notes from the famous mathematical lectures delivered by Emmy Noether (1882–1935), Karl Grandjot (1900–1979), Pavel Sergeevich Aleksandrov (1896–1982) and Bartel Leendert van der Waerden (1903–1996). Jarník's notebooks were discovered by Jindřich Bečvář in 2004.

Walter BERGWEILER

Christian–Albrechts–Universität zu Kiel (Kiel, Germany)

Real zeros of solutions of linear differential equations

Let A be a transcendental entire function of finite order. We show that if the differential equation $w'' + Aw = 0$ has two linearly independent solutions with only real zeros, then the order of A must be an odd integer or one half of an odd integer. Moreover, A has completely regular growth in the sense of Levin and Pfluger.

The results are joint work with Alexandre Eremenko and Lasse Rempe.

Bappaditya BHOWMIK

Indian Institute of Technology Kharagpur (Kharagpur, India)

Existence of nonzero poles produce nontrivial lower bounds for coefficients of certain harmonic univalent functions

Let $A_H(p)$ be the class of all sense preserving complex valued harmonic functions f in the open unit disc of the complex plane having a simple pole at $z = p \in (0,1)$ with the normalizations $f(0) = f_z(0) - 1 = 0$. We first obtain a sufficient condition for univalence of such functions. Next, we consider the class $S_H^0(p)$ which consists of all functions belonging to $A_H(p)$ that are univalent with the additional normalization $f_{\bar{z}}(0) = 0$. We discuss about the class $S_H^0(p)$ and certain subclasses of it in the view point of geometric function theory. Interestingly, consideration and study on these subclasses of $S_H^0(p)$ yield nontrivial lower bounds for coefficients of these functions and as consequences, we pose conjectures on general bounds of these coefficients. This talk is based on the following articles:

(i) B. Bhowmik, S. Majee, *On harmonic univalent mappings with nonzero pole*, J. Math. Anal. Appl. **482** (2020), no. 1, 13 pp.
(ii) B. Bhowmik, S. Majee, *On the coefficients of certain subclasses of harmonic univalent mappings with nonzero pole*, Bull. Braz. Math. Soc. **52** (2021), no. 4, 1041–1053.

Filippo BRACCI

Università di Roma "Tor Vergata" (Roma, Italia)

Abstract boundaries and applications

In this talk I will survey on some abstract boundary and related constructions which can be constructed using invariant metrics on complex manifolds (mainly our focus will be on domains and Gromov's boundary) and applications such as extension of biholomorphisms and Denjoy-Wolff type theorem.

Teodor BULBOACĂ

Bebeş-Bolyai University (Cluj-Napoca, Romania)

Subordination properties and initial coefficients bounds for a subclass of convex functions

The aim of this investigation is to introduce and study a novel class of analytic functions associated with convex functions in the open unit disc. Relationships of this class with other subclasses of analytic functions are derived. Further, different results for the class of such functions and many new interesting properties are obtained.

Motivated essentially by the works [1, 2, 3] we will define a new subclass of analytic functions, and we will present its connections with some other existing subclasses, as follows:

Definition 1. A function $f \in \mathcal{A}$ belongs to the subclass $\mathcal{S}_{\mathcal{CV}}$ if

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right) > \left|\frac{zf''(z)}{f'(z)}+f'(z)-1\right|, \quad z \in \mathbf{U} := \{z \in \mathbf{C} : |z| < 1\}.$$

We note that the above definition implies the image $f(\mathbf{U})$ of all the functions $f \in \mathcal{S}_{\mathcal{CV}}$ is a convex domain, and the main results are the next ones:

Theorem 2. (i) *If $f \in \mathcal{S}_{\mathcal{CV}}$, then*

$$f'(z) \prec \frac{2\sqrt{z}}{1-z}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^{-1} = \frac{1}{{}_2F_1\left(1,1;\frac{3}{2};\frac{z}{z-1}\right)}, \quad z \in \mathbf{U},$$

where ${}_2F_1(a,b;c;z)$ is the Gaussian hypergeometric function.

(ii) *If $f \in \mathcal{S}_{\mathcal{CV}}$ with $f''(0) \neq 0$, then*

$$\left|\arg f'(z)\right| < \frac{\pi}{4}, \quad z \in \mathbf{U}.$$

(iii) *If* $f \in \mathcal{S}_{\mathcal{CV}}$*, then*

$$\frac{r\,[1-(1-2\lambda)r]}{1+r} \le |f(z)| \le \frac{r\,[1+(1-2\lambda)r]}{1-r}, \quad |z| \le r < 1,$$

where $\lambda = 2/\pi$*. The above two bounds are the best possible.*

Theorem 3. *If* $f \in \mathcal{S}_{\mathcal{CV}}$ *has the form*

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \quad z \in \mathbf{U},$$

then

(i)

$$|a_2| \le \frac{1}{3}, \quad |a_3| \le \frac{2}{9}, \quad |a_4| \le \frac{1}{6}.$$

(ii) *The logarithmic coefficients of* f *satisfy the inequalities*

$$|\gamma_1| \le \frac{1}{6}, \quad |\gamma_2| \le \frac{1}{9}, \quad |\gamma_3| \le \frac{1}{12}.$$

(iii) *The second Hankel determinant satisfies the inequality*

$$\left|a_2 a_4 - a_3^2\right| \le \frac{4}{81}.$$

References

[1] N. Hameed Mohammed, E. A. Adegani, T. Bulboacă, N. E. Cho, *A family of holomorphic functions defined by differential inequality*, Math. Inequal. Appl. **25** (2022), 27–39.

[2] W. Ma, D. Minda, *Uniformly convex functions*, Ann. Polon. Math. **57** (1992), no. 2, 165–175.

[3] F. Rønning, *Uniformly convex functions and a corresponding class of starlike functions*, Proc. Amer. Math. Soc. **118** (1993), 189–196.

Jacek CHUDZIAK

University of Rzeszów (Rzeszow, Poland)

On differentiable solutions of the translation equation

Inspired by a question stated in [2], we deal with a problem of approximation of continuous solutions $F : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ of the translation equation

$$F(F(x,s),t) = F(x,s+t) \quad \text{for} \quad x,s,t \in \mathbf{R},$$

by the differentiable solutions of this equation.

References

[1] Z. Moszner, *On the differentiability of the solutions of translation equation*, Aequat. Math. **96** (2022), 723–739.

Zoltán Ernő CSAJBÓK

University of Debrecen (Debrecen, Hungary)

Rough real functions and intuitionistic fuzziness

In the mid-1990s, Z. Pawlak, based on the rough set theory (RST) [6, 7, 9], initiated studying the rough calculus of real functions [8]. Rough real functions are real functions attached to a special Cartesian coordinate system. The values of real functions are categorized via the x and y axes.

Papers [3, 5] connected the rough real functions and the intuitionistic fuzzy sets. However, this connection only covered the real functions that take values in the unit interval. Later, in [4], more realistic real functions have connected with intuitionistic L-fuzzy sets [1, 2]. The boundary region on the rough real function side, and the hesitancy function on the intuitionistic L-fuzzy set side, are extremely important for rough real functions. These two regions are a connecting link, both semantically and syntactically, between the two otherwise distant areas.

The paper will present how rough real functions connect with intuitionistic fuzziness, and analyze their mutual interactions.

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Suman DAS

Indian Institute of Technology Ropar (Rupnagar, India)

Riesz-Fejér type inequalities for harmonic mappings

A celebrated result in the theory of Hardy spaces (on the unit disk) is the Riesz-Fejér inequality of the form

$$\int_{-1}^{1} |f(x)|^p \mathrm{d}x \leq \frac{1}{2}\int_{0}^{2\pi} |f(\mathrm{e}^{\mathrm{i}\theta})|^p \mathrm{d}\theta, \quad 0 < p < \infty. \tag{1}$$

This has an elegant geometric description: if the unit disk is mapped conformally onto the interior of a rectifiable Jordan curve C, the image of any diameter has length at most half the length of C. Frazer [2] notably refined this inequality by comparing the integral mean along a circle to the same along a pair of diameters. Recently the sharp form of inequality (1) for the harmonic Hardy space is obtained in [3] (for $1 < p \leq 2$) and [4] (for $p > 2$). Here we shall discuss a harmonic analogue of Frazer's result as well as its consequences, including the generalization of a classical inequality of Hilbert. The content of this talk is primarily based on the article [1].

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Katarzyna DEMS-RUDNICKA, Izabela JÓŹWIK

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Królowa Nauk na co dzień – o cyklu wykładów popularyzujących matematykę wśród młodzieży

W referacie omówimy cykl wykładów popularyzujących matematykę wśród uczniów szkół ponadpodstawowych i studentów. Każdy z wykładów prezentuje różne zagadnienia matematyczne, dobrane pod kątem zastosowań w życiu codziennym np. zagadnienia optymalizacyjne z wykorzystaniem rachunku różniczkowego funkcji jednej zmiennej rzeczywistej (optymalny dobór kształtu działki, czy kształtu okien w budowanym domu), teoria grafów (dobór najlepszej trasy podróży), ciąg Fibonacciego (jego obecność w przyrodzie, w ogrodzie). Każdy wykład stanowi osobną całość i może być prezentowany samodzielnie, a jednocześnie wszystkie razem układają się w logiczny cykl, który jest wciąż rozbudowywany i udoskonalany.

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Renata DŁUGOSZ, Bartosz BARTOSZEK

Lodz University of Technology (Lodz, Poland)

On a multiplicative distribution of functions in complex plane

During the lecture there are considered two kinds of functional symmetry, i.e., (1)-power symmetry and the (-1)-power symmetry, in a rich subfamily $\mathcal{B}$ of the family of all complex valued functions on symmetric set $\mathcal{G} \subset \mathbf{C}$. The first one defines the values $f(-z)$ as identical with

$f(z)$ and the other as inverse of $f(z)$ in $\mathcal{G}$. The announced notions allow a unique decomposition of functions $f \in \mathcal{B}$ onto a product of two factors f_1, f_{-1} having the (1)- and the (-1)-power symmetry property, respectively. In the talk there are also given examples of such partitions $f = f_1 \cdot f_{-1}$, for various $f \in \mathcal{B}$, a solution of the problem of invariance the above functional symmetries with respect to some one- and two-arguments functions operations and two applications of the mentioned function decomposition.

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Renata DŁUGOSZ, Piotr LICZBERSKI

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Fekete-Szegö problem for Bavrin's holomorphic functions and biholomorphic mappings in $\mathbf{C}^n$

In the course of the lecture, certain properties of functions $f : \mathcal{G} \to \mathbf{C}$ holomorphic on bounded complete n-circular domains $\mathcal{G} \subset \mathbf{C}^n$ will be considered. Such functions have a unique expansion into series of m-homogeneous polynomials $f(z) = \sum_{m=0}^{\infty} Q_{f,m}(z), z \in \mathcal{G}$. For this reason, the concept of Minkowski balance $\mu_{\mathcal{G}}(Q_m)$ [5] of m-homogeneous polynomials Q_m is very useful in characterizing Bavrin's functions [1].

During the first part of the lecture, for functions from a Bavrin's family, will be presented a sharp upper estimate of the Minkowski balance $\mu_{\mathcal{G}}(Q_2)$ of the 2-homogenous polynomials $Q_2 = Q_{f,2} - \lambda\,(Q_{f,1})^2\,, \lambda \in \mathbf{R}$, [2]. This is a generalization of the Fekete-Szegö problem defined in [3], solved in [4] for a class of univalent functions of a complex variable, to the s.c.v. case.

In the second part of the lecture, an application of the above result in solving generalized Fekete-Szegö type problem for the mappings $F : \mathbf{B}^n \to \mathbf{C}^n$ transforming the Euclidean ball $\mathbf{B}^n$ onto close-to-starlike domains $F(\mathbf{B}^n)$ will be presented. For such mappings a sharp upper bound will be given for a certain quantity dependent on Frechet differentials $\mathrm{D}^2F(0)(z^2), \mathrm{D}^3F(0)(z^3),\ z \in \mathbf{B}^n$.

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Renata DŁUGOSZ, Monika LINDNER, Piotr OSTROWSKI

Lodz University of Technology (Lodz, Poland)

Consequences of moving away from stationary math teaching in high schools during pandemic

During the lecture the results showing the scale of the reduction in the knowledge of high school graduates as a consequence of remote teaching will be presented. The lecture is based on data from one of the technical faculties of the Lodz University of Technology. The organizational and didactic activities implemented by the authorities of the faculty and lecturers to improve the existing situation will also be discussed.

Stanisław DOMORADZKI

Institute of History, University of Rzeszów (Rzeszow, Poland)

The importance of the Commission of National Education for the development of mathematics in Poland in the years 1773-1794.

In the talk, we will recall how an outstanding educational organization, the Commission of National Education, was established in Poland during the country's political and economical disintegration in the last three decades of the 18th century. We will present its influence on the shaping of modern mathematics in Poland.

Michael DORFF

Brigham Young University (Provo, Utah)

Zeros of a one-parameter family of harmonic trinomials

It is well known that complex harmonic polynomials of degree n may have more than n zeros. In this talk, we discuss a one-parameter family of harmonic trinomials and show that the number of zeros depends on the parameter. Our proof heavily utilizes the Argument Principle for Harmonic Functions and involves finding the winding numbers about the origin for a family of hypocycloids.

Jacek DZIOK

University of Rzeszów (Rzeszow, Poland)

Classes of meromorphic harmonic functions defined by weak subordination

A continuous function $f = u+iv$ is a complex valued harmonic function in a domain D if both u and v are real harmonic in D. If D is the exterior of the unit disc i.e. $\mathbf{D} := \{z \in \mathbf{C} : |z| > 1$, then we say that f is meromorphic harmonic function. Hengartner and Schober [1] showed that meromorphic harmonic, orientation preserving, univalent mapping f, satisfying $f(\infty)=\infty$, must admit the representation $f(z)=h(z)+\overline{g(z)}+A\log|z|$, where $A\in\mathbf{C}$ and h, g are functions analytic in $\mathbf{D}$. We remove the logarithmic singularity in by letting $A=0$ and focus on the family $\Sigma_{\mathcal{H}}$ of meromorphic harmonic orientation preserving univalent mappings of the form $f(z)=h(z)+\overline{g(z)}$, where

$$h(z)=z+\sum_{n=1}^{\infty}a_n z^{-n},\quad g(z)=\sum_{n=1}^{\infty}b_n z^{-n},\qquad z\in\mathbf{D}.$$

Motivated by Mauir [2] we define a weak subordination for complex-valued functions in $\mathbf{D}$. A complex-valued function f in $\mathbf{D}$ is said to be *weakly subordinate* to a complex-valued function F in $\mathbf{D}$, and we write $f\preceq F$, if $f(\infty)=F(\infty)$ and $f(\mathbf{D})\subset F(\mathbf{D})$. If F is univalent in $\mathbf{D}$, then $f\preceq F$ if and only if there exists a complex-valued function ω which maps $\mathbf{D}$ into oneself with $\omega(\infty)=\infty$ such that $f(z)=F(\omega(z)),\ z\in\mathbf{D}$.

The object of the present talk is to define and study some subclasses of the class $\Sigma_{\mathcal{H}}$ defined by the weak subordination.

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Mark ELIN

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Filtration of generators and an inverse Fekete-Szegö problem

In this talk we present problems belonging to (a) dynamic system and to (b) geometric function theory, in their correlation. In the first part of the talk, we study the problem of characterizing membership of normalized holomorphic functions of the disk to the class of infinitesimal generators and their sectorial and analytic extension. Inter alia presenting results can be applied to dynamics of the corresponding semigroups using filtrations of the class of infinitesimal generators.

In the second part we introduce and study a question that can be interpreted as ‘an inverse Fekete–Szegö problem’. This problem links to the first part of the talk. We introduce new filtration classes using the non-linear differential operator

$$\alpha\cdot\frac{f(z)}{z}+\beta\cdot\frac{zf'(z)}{f(z)}+(1-\alpha-\beta)\cdot\left[1+\frac{zf''(z)}{f'(z)}\right],$$

and establish certain properties of these classes. Sharp upper bounds of the absolute value of the Fekete-Szegö functional over some filtration classes are found. We also present open problems for further study.

The talk is based on joint works [1, 2, 3].

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Szymon GŁĄB

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On strong algebrability of families of non-measurable functions of two variables

Let $\mathcal{L}$ be a commutative algebra, $A \subseteq \mathcal{L}$ and let κ be a cardinal number. We say that A is *strongly κ-algebrable* if $A \cup \{0\}$ contains a κ-generated subalgebra B that is isomorphic to a free algebra.

In our work we present improvements of the results presented in the article by Tomasz Natkaniec [1] in the direction of strong algebrability. We deal with the algebra $F(\mathbf{R}^2, \mathbf{R})$ of all real functions defined on the real plane $\mathbf{R}^2$. Among other we prove that

- Assuming CH, the family of all sup-measurable functions that are not measurable is strongly $2^{\mathfrak{c}}$-algebrable.
- Assuming CH, the family of all weakly sup-measurable functions that are neither sup-measurable nor measurable is strongly $2^{\mathfrak{c}}$-algebrable.
- The family of all non-measurable separately measurable functions is strongly $2^{\mathfrak{c}}$-algebrable.

Since the cardinality of $F(\mathbf{R}^2, \mathbf{R})$ is $2^{\mathfrak{c}}$, our results are optimal.

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Marek GOLASIŃSKI

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The exceptional Lie group F_4: its applications and generalizations

The octonions $\mathbf{O}$ are non-commutative and non-associative and satisfy a weaker form of associativity; namely, they are alternative and power associative. Octonions have applications in fields such as string theory, special relativity and quantum logic and are related to exceptional structures in mathematics:

Classification Theorem. *Every compact, connected, simply-connected Lie group is a product of finitely many compact, connected, simply-connected simple Lie groups each of which is isomorphic to exactly one of the following:*

- *the symplectic group* $\mathrm{Sp}(n)$ *for* $n \geq 1$*;*
- *the special unitary group* $\mathrm{SU}(n)$ *for* $n \geq 3$*;*
- *the spin group* $\mathrm{Spin}(n)$ *for* $n \geq 7$*.*

or one of the five exceptional Lie groups $G_2 \subseteq F_4 \subseteq E_6 \subseteq E_7 \subseteq E_8$*.*

Recall that $G_2 = \mathrm{Aut}_{\mathbf{R}}(\mathbf{O})$, the group of $\mathbf{R}$-linear automorphisms of $\mathbf{O}$.

To define the group F_4, consider the $\mathbf{R}$-Jordan algebra $\mathrm{Herm}_3(\mathbf{O})$ of 3×3-Hermitian matrices over $\mathbf{O}$ equipped with the binary operation

$$X \circ Y := \frac{1}{2}(XY + YX)$$

for $X, Y \in \text{Herm}_3(\mathbf{O})$. Then, the automorphism group $\text{Aut}_{\mathbf{R}}(\text{Herm}_3(\mathbf{O}), \circ)$ is denoted by F_4. Furthermore, the group F_4 is isomorphic to the isometry group of the Cayley plane $\mathbf{OP}^2$.

The talk grew out of our desire to develop techniques on octonians $\mathbf{O}(K)$ over a real closed field K and present an explicit diagonalization of 3×3-Hermitian matrices over $\mathbf{O}(K)$ via the group $F_4(K)$.

For the an action of $F_4(K)$ on the K-Cayley plane $\mathbf{O}(K)\mathbf{P}^2$ and the stabilizer $\text{Spin}(9, K)$ of the matrix $E_{11} \in \mathbf{O}(K)\mathbf{P}^2$, the main result states:

Theorem. (1) *If K is a real closed field, then any matrix $X \in \text{Herm}_3(\mathbf{O}(K))$ can be transformed to a diagonal form by some element $\varphi \in F_4(K)$.*

(2) *If K is a formally real Pythagorean field, then the map $F_4(K) \to \mathbf{O}(K)\mathbf{P}^2$ given by $\varphi \mapsto \varphi(E_{11})$ for $\varphi \in F_4(K)$ yields a bijection*

$$F_4(K)/\text{Spin}(9, K) \xrightarrow{\approx} \mathbf{O}(K)\mathbf{P}^2.$$

Aleksandra HUCZEK

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The Wolff-Denjoy type theorem for semigroups in geodesic spaces

One of the most important theorem which relates to dynamics of nonlinear mappings is the Wolff-Denjoy theorem. The classical version of this theorem [6] asserts that if $f : \Delta \to \Delta$ is a holomorphic mapping of the unit disc $\Delta \subset \mathbf{C}$ without a fixed point, then there is a point $\xi \in \partial\Delta$ such that the iterates f^n converge locally uniformly to ξ on Δ. The above theorem has been generalized over the years in different directions (see e.g. [1, 2, 3, 4]).

The aim of this talk is to extend the Wolff-Denjoy theorem to the case of one-parameter continuous semigroups of nonexpansive (i.e. 1-Lipschitz) mappings acting on a complete locally compact geodesic spaces. In particular, our results applies to strictly convex bounded domains in finite dimensional (real or complex) space with respect to large class of metrics including Kobayashi's and Hilbert's metrics.

This is a joint work with Andrzej Wiśnicki.

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KAPIL JAGLAN

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On Cesàro means for harmonic mappings

Let $f(z) = z + \sum_{k=2}^{\infty} a_k z^k \in \mathcal{S}$, the class of analytic univalent functions in unit disk $\mathbf{D}$, the n^{th} section/partial sum $s_n(f)$ is defined by

$$s_n(f) = z + \sum_{k=2}^{n} a_k z^k.$$

Dealing with a function's series representation might be difficult at times, although dealing with its partial sums is much easier. But the partial sums of univalent functions need not be univalent. In [6], Szegö showed that $s_n(f)$ is univalent in the disk $|z| < 1/4$ for all $n \geq 2$, and the number $1/4$ cannot be replaced by a larger one. Later, Jenkins [2] improved the estimate, however, the exact radius of univalence r_n of $s_n(f)$ remains an open problem for $f \in \mathcal{S}$.

Another interesting problem in this line was to study the geometry preserving polynomial approximation of univalent functions from various geometric subclasses of $\mathcal{S}$. Fejér [1] and Ruscheweyh [3] discussed the geometry preserving properties of the partial sums $S_n^{\alpha}(f)$, the n^{th} Cesàro means of order $\alpha \geq 0$ and $V_n(f)$, the n^{th} *De la Vallée Poussin means* of function f. Considering the harmonic case, recently Sairam Kaliraj [4] studied the geometric nature of $V_n(f)$ means for some subclasses of $\mathcal{S}_H^0$, the class of all sense-preserving normalized univalent harmonic mappings. Here [5] we shall discuss the geometry preserving properties of the partial sums $S_n^{\alpha}(f)$ for univalent harmonic mapping f, when the range of f is a convex, starlike, close-to-convex, or convex in one direction domain. We also determine the radius of fully starlikeness (respectively, fully convexity) of $S_2^{\alpha}(f)$, when $f \in \mathcal{S}_H^0$.

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IZABELA JÓŹWIK, MAŁGORZATA TEREPETA

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100 lat twierdzenia Banacha o punkcie stałym. Część 1 i 2.

24 czerwca 1920 roku Stefan Banach przedstawił swoją rozprawę doktorską *O operacjach na zbiorach abstrakcyjnych i ich zastosowaniach do równań całkowych.* Na Wydziale Filozoficznym Uniwersytetu Jana Kazimierza we Lwowie zdał egzaminy doktorskie z matematyki, fizyki i filozofii. W styczniu 1921 roku został doktorem. Rok później opublikował wyniki swojego doktoratu

w Fundamenta Mathematicae. Wśród nich znalazło się twierdzenie znane obecnie pod nazwą twierdzenie Banacha o punkcie stałym lub zasada kontrakcji Banacha. Jest to jedno z najbardziej znanych twierdzeń matematycznych, jedno z licznych z nazwiskiem Banacha w nazwie.

W 2022 roku obchodzimy stulecie publikacji twierdzenia Banacha o punkcie stałym. W referacie chcemy przedstawić jego najważniejsze modyfikacje i uogólnienia, pewne nieliniowe warunki zwężania uogólniające klasyczną definicję odwzorowania zwężającego podaną w 1922 roku przez Stefana Banacha, twierdzenie odwrotne oraz pewne zastosowania.

Nie jest możliwe przedstawienie pełnej informacji o tym, co zostało napisane w czasie minionych stu lat o twierdzeniu Banacha o punkcie stałym, dlatego głównym celem naszego wystąpienia jest uporządkowanie wiedzy na ten temat oraz przedstawienie bibliografii, do której mogą się odwołać wszyscy zainteresowani tymi zagadnieniami.

A. SAIRAM KALIRAJ

Indian Institute of Technology Ropar (Rupnagar, India)

Growth of harmonic functions and its applications

Hardy and Littlewood's seminal works on the growth of analytic functions contain the comparison of the integral means $M_p(r,f)$, $M_p(r,f')$, $M_q(r,f)$. For a complex-valued harmonic function f in the unit disk, using the notation $|\nabla f| = (|f_z|^2 + |f_{\bar{z}}|^2)^{1/2}$ we explore the relation between $M_p(r,f)$ and $M_p(r,\nabla f)$. We show that if $|\nabla f|$ grows sufficiently slowly, then f is continuous on the closed unit disk and the boundary function satisfies a Lipschitz condition. We also prove that for $1 \le p < q \le \infty$, it is possible to give an estimate on the growth of $M_q(r,f)$ whenever the growth of $M_p(r,f)$ is known. We notably obtain Baernstein type inequalities for the major geometric subclasses of univalent harmonic mappings such as convex, starlike, close-to-convex, and convex in one direction functions. Some of these results are sharp. A growth estimate and a coefficient bound for the whole class of univalent harmonic mappings are given as well. The content of this talk is mostly based on the article [1].

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Adel KHALFALLAH

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Schwarz-Pick lemma for harmonic and hyperbolic harmonic functions

In this talk, we establish some inequalities of Schwarz-Pick type for harmonic and hyperbolic harmonic functions on the unit ball and we disprove a recent conjecture of Liu [Schwarz-Pick Lemma for Harmonic Functions, International Mathematics Research Notices, 2021]. Connection to Khavinson's conjecture will also be discussed.

Miljan KNEŽEVIĆ

University of Belgrade, Faculty of Mathematics (Belgrade, Republic of Serbia)

Some properties of the quasi-conformal diffeomorphisms of the unit disc

We will give a new idea how to consider some obtained distortion results of the class of HQC diffeomorphisms of the unit disc $\mathbf{D}$ in order to get new Schwarz-Pick type results for that class of functions. In particular, we will give the answers to many questions concerning those mappings which are related to the determination of different properties that are of essential importance for validity of the results such as those that generalize famous inequalities of the Schwarz-Pick type. The approach used is geometrical in nature.

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[1]*University of Warmia and Mazury in Olsztyn* (Olsztyn, Poland)
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On some computational and mathematical applications

The proposed paper is devoted to the description of some computational and mathematical methods and their application in medicine. By the use of modeling approach we study some phenomena related to modern diseases. Preliminaries to the research area are provided. Theoretical foundations of the used models are presented. The results of the conducted numerical experiments are shown, which are commented from a medical point of view.

Sushil KUMAR

Bharati Vidyapeeth's College of Engineering (New Delhi, India)

Properties of certain functions associated with the right half plane its applications and generalizations

We discuss some techniques for computation of estimates on Hermitian-Toeplitz determinants, Hankel determinants as well as some subordination inclusions for certain starlike functions which are associated with the right half plane and defined on open unit disk.

AMS Subject Classification: Primary 30C45, 30C50, 30C80

Virendra KUMAR

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Recent development on coefficient functionals in univalent function theory

The coefficients functionals for functions in various subclasses of univalent functions have been most studied topics in the recent years. It becomes, therefore, interesting when it comes to investigate sharp bound on them. In the present lecture is a talk about recent developments on Hankel, Hermitian-Toeplitz determinants and some related open problems.

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[2] *The University College of Applied Sciences in Chełm* (Chełm, Poland)

Generalized Fekete-Szegö functionals

Let $\mathcal{S}$ be the class of all injective and holomorphic functions f in the unit disk $\mathbf{D}$ with classical normalization $f(0) = 0 = f'(0) - 1$. The talk deals with the following coefficient functionals

$$\mathcal{S} \ni f \mapsto |\,\mathrm{c}_3(f) - \lambda\,\mathrm{c}_2(f)^2| - \mu|\,\mathrm{c}_2(f)|, \quad \lambda \in \mathbf{C},\, \mu \in \mathbf{R}, \tag{1}$$

where $\mathrm{c}_n(f)$ denotes the n-th coefficient of $f \in \mathcal{S}$. These functionals reduce to the classical Fekete-Szegö functionals for $\mu = 0$. The case where $\mu = 1$ is of special interest due to deep relationship to successive initial coefficients problems. To be more specific:

- If $\lambda = 0$, then the functional (1) corresponds to the successive coefficients for $f \in \mathcal{S}$.
- If $\lambda = 1/2$, then the functional (1) corresponds to the successive logarithmic coefficients for $f \in \mathcal{S}$.
- If $\lambda = 3/2$, then the functional (1) corresponds to the successive coefficients of the inverse function of $f \in \mathcal{S}$.
- If $\lambda = 2$, then the functional (1) corresponds to the successive logarithmic coefficients of the inverse function of $f \in \mathcal{S}$.

The sharp lower and upper estimations of the functional (1) are discussed and the extremal functions are determined for certain $\lambda \in \mathbf{C}$ provided $\mu = 1$. The results are applicable to some standard subclasses of $\mathcal{S}$.

See Keong LEE

School of Mathematical Sciences, Universiti Sains Malaysia (Penang, Malaysia)

The Bohr operator on analytic functions

The Bohr operator $\mathcal{M}_r$ for a given analytic function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and a fixed z in the unit disk, with $|z| = r$, is given by

$$\mathcal{M}_r(f) = \sum_{n=0}^{\infty} |a_n||z^n| = \sum_{n=0}^{\infty} |a_n| r^n.$$

Some inequalities involving this operator acting on certain analytic functions and its section will be discussed.

ADAM LIGĘZA

University of Warsaw (Warsaw, Poland)

Hamiltonians of Painlevé V equation

In this talk I will speak about Painlevé equations, especially about the fifth Painlevé equation P_V. I will discuss three different Hamiltonians and Hamiltonian systems connected with P_V (that are Okamoto's Hamiltonian, rational Hamiltonian and KNY Hamiltonian) and a method how they can be matched with usage of algebraic geometry tools. I will show how that can be done by matching surface roots on the level of Picard lattice. Moreover I will check whether our matching is canonical.

This is a joint work with Galina Filipuk, Anton Dzhamay and Alexander Stokes.

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Monika LINDNER, Renata DŁUGOSZ

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Zajęcia z arytmetyki finansowej w kontekście zmieniającej się sytuacji ekonomicznej Polski

W referacie przedstawione zostaną próby uwzględnienia w zadaniach z arytmetyki finansowej zmian w otaczającym nas środowisku ekonomicznym. Wspomniane będą w szczególności ujemne stopy procentowe w Szwajcarii, wysoka inflacja i obligacje indeksowane wskaźnikiem inflacji.

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Violetta LIPIŃSKA

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The form of a competence exam at the Master's degree studies at the Lodz University of Technology

The Lodz University of Technology is the only university in Poland that comprehensively confirms the quality of the awarded qualifications, and thus the quality of the university's diploma. In addition to the diploma exam, an original model of the competence exam was developed based on the case study methodology.

This exam verifies the student's achievement of aggregate key competences specified for the study program. It consists in analyzing descriptions of selected, specific events in the field of studies with all their complexities and difficulties.

The result of the competence exam becomes part of the grade for studies, in accordance with § 41 it. 11 of the Study Regulations.

The structure of the competence exam for second-cycle studies (Master's degree studies) will be discussed. The *Choosing an investment portfolio* exam case will be presented and the problems encountered by the students while solving it will be discussed.

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Abdallah LYZZAIK

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Moduli of doubly connected domains under univalent harmonic maps

Recently, a pioneering proof of Nitsche's conjecture [3] by Iwaniec, Kovaliv and Innonen [2] on univalent harmonic mappings between two annuli led the authors to raise in [1] the following question: For which values s, t, $1 < s, t < \infty$, does there exist a harmonic homeomorphism

$f : \mathcal{T}(s) \to \mathcal{T}(t)$, where $\mathcal{T}(.)$ is a Teichmüller domain? By restricting ourselves to harmonic homeomorphisms symmetric about the real axis, we establish a two-fold purpose: (a) a solution for this problem by using the theory of extremal length, and (b) test Conjecture 1.4 of [1] regarding the moduli of the doubly-connected domains related by harmonic homeomorphisms in light of our results. The talk concludes with some relevant interesting questions.

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Marek MAŁOLEPSZY

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Rola kontekstu w kształceniu matematycznym na studiach technicznych

Matematyka jest jednym z podstawowych przedmiotów dostarczającym niezbędnych kompetencji do dalszego studiowania, stanowiącym podstawę kształcenia na studiach technicznych. Jednak żeby była ona efektywnie wykorzystywana na studiach i w pracy zawodowej, muszą zajść określone warunki. W szczególności, student musi znać oraz umieć posługiwać się narzędziami matematycznymi, a ponadto musi potrafić zastosować je do rozwiązania postawionego przed nim problemu. W wystąpieniu przedstawiona zostanie rola jaką odgrywa kontekst w procesie kształcenia matematycznego przyszłych inżynierów, jego wpływ na motywację, rozumienie oraz transfer dydaktyczny. Omówione będą wyniki badania przeprowadzonego wśród nauczycieli akademickich, dotyczącego wykorzystania matematyki na przedmiotach kierunkowych na studiach technicznych.

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Jacek MARCHWICKI

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Achievement sets of reciprocals of a complete sequences

The purpose of this talk is to introduce readers with achievement set, that is the set of subsums of the series $\sum_{n=1}^{\infty} x_n$, more precisely

$$A(x_n) = \left\{ \sum_{n=1}^{\infty} \varepsilon_n x_n : (\varepsilon_n) \in \{0,1\}^{\mathbf{N}} \right\} = \left\{ \sum_{n \in A} x_n : A \subset \mathbf{N} \right\}.$$

Brown introduced the notion of complete sequences, that is $(y_n) \subset \mathbf{N}^{\mathbf{N}}$ is complete if $A(y_n) = \mathbf{N}$ or equivalently $y_n \leq 1 + \sum_{k=1}^{n-1} y_k$ for each $n \in \mathbf{N}$. Jones considered sequences of reciprocals of complete sequences that is $(1/y_n)$. He asked what can be said about $A(1/y_n)$ with suggesting that $A(1/y_n)$ is a finite sum of closed intervals for absolutely convergent sequences. We give the answer for his question by showing that the notion of completeness if somehow connected with Kakeya's conditions.

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Joanna MARKOWICZ

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On some geometric properties of interpolation spaces obtained with the general discrete K-method

One of the problems appearing in the researches of Banach spaces is studying conditions which preserve a given geometric property of a Banach space under some constructions, such as interpolation spaces.

Among many well-known geometric properties of Banach spaces we may enumerate uniform convexity and Opial properties.

The aim of the presentation is to give the conditions under which uniform convexity and Opial properties are preserved in interpolation spaces.

We consider interpolation spaces whose construction is based on the general discrete K-method of interpolation. In that construction we use an abstractive space with an unconditional basis.

The presentation will start from giving the short characerization of the method. Subsequently, there will be given the result that gives conditions for an interpolation space to be uniformly convex. Furthermore, the conditions that guarantee an interpolation space to have the Opial property and the uniform Opial property will be presented.

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Miodrag MATELJEVIĆ

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Boundary behaviour of partial derivatives for solutions to certain Laplacian-gradient inequalities and spatial QC maps

The subject of our study are quasiconformal mappings in the plane and space between smooth domains which satisfy a inequality which we call the Laplacian-gradient inequality (in the literature it was also called Poisson's differential inequality). As an application, we get some results which we can considered as spatial versions of Kellogg's theorem. We also outline some results of this type for harmonic maps and maps which satisfy PDE of second order.

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Nonstationary fractional-order systems

Fractional-order systems (FOS) are dynamical systems that can be modelled by fractional differential equations involving derivatives of non-integer order. FOS are useful in investigating memory effect and hereditary properties of various materials and processes. While linear FOS represent a fairly well investigated field of research, relatively few papers deal with nonstationary linear FOS described by fractional differential equations (FDEs) with variable coefficients.

Meanwhile, a number of real-life systems and processes can be described by linear FDEs with variable coefficients. Linear differential equations with variable coefficients arise in a natural way when modeling RLC-circuits with variable capacitance or inductance. With the advent of electronic components like super-capacitors (also called ultracapacitors) and fractances, one should employ fractional differential equations for circuit models. This provides motivation for research on FDEs with variable coefficients and related control problems.

In the recent years a number of papers have been devoted to solutions of the systems of FDEs with variable coefficients and their control. In [4] explicit solutions for the linear systems of initialized FDEs are obtained in terms of generalized Peano–Baker series. Linear systems of FDEs with variable coefficients and their state-transition matrices are also discussed in [5].

In this paper the initial value problem for linear systems of FDEs with variable coefficients involving Riemann–Liouville and Caputo derivatives is investigated. For these systems solution of initial-value problem is derived in terms of the generalized Peano–Baker series and time-optimal control problem is formulated. The optimal control problem is treated from convex-analytical viewpoint. Necessary and sufficient conditions for time-optimal control similar to that of Pontryagin's maximum principle are obtained. The research is a further development of the approach consisting in extension of the Pontryagin maximum principle to fractional-order systems [2, 3, 6].

Further, differential games described by the systems of linear FDEs with variable coefficients involving Riemann–Liouville and Caputo derivatives are also examined. The game problem is treated from convex-analytical viewpoint using the method of resolving functions. On the basis of the resolving functions method sufficient conditions for the finite-time game termination from given initial states are derived.

Theoretical results are supported by illustrative examples.

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ANDRZEJ MICHALSKI

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Some estimates for coefficients of bounded analytic functions

Let Δ stand for the open unit disc in the complex plane $\mathbf{C}$ and denote by B the class of all analytic functions

$$\omega(z) = \sum_{n=0}^{\infty} c_n z^n,$$

where $c_n \in \mathbf{C}$, defined on Δ satisfying $|\omega(z)| \leq 1$.

Properties of the class B are widely used in complex analysis, in particular, such functions play an important role in the theory of planar harmonic mappings (see e.g. [1]). In fact, our researches of this topic (see e.g. [2]) gave us motivation to study properties of bounded analytic functions and by use of Schur's theorem [4] led us to several remarks on some coefficient expressions for functions of the class B ([3]).

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KAROLINA MROCZYŃSKA

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Reasoning and argumentation of a student with Asperger Syndrome in the mathematics lessons

The aim of the paper is to analysis the reasoning and argumentation of a student with Asperger Syndrome in mathematics lessons. Based on the analysis of the high school exam in mathematics and experiences from the lessons, mathematical thinking –"winding path" to solve the mathematical problem will be presented. For a student with Asperger Syndrome, the subject of the task itself may be the first obstacle. Reasoning and argumentation is the most important and at the same time the most difficult general goal to be achieved defined in the core curriculum. Reasoning or logical thinking is a way of understanding the world, its elements and relations between them. This ability has a significant impact on the social functioning which is a special area in the life of a person with Asperger Syndrome.

Anna MURANOVA

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Spectrum of discrete Laplacian over an ordered field

We consider discrete normalized Laplacian for the finite graphs, whose edge-weights belong to an arbitrary real-closed ordered field. We show that eigenvalues of Laplacian always belong to the same field. Moreover, estimates of the eigenvalues in terms of number of vertices as well as in terms of isoperimetric constants will be presented.

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Saima MUSTAFA

PMAS, Arid Agriculture University (Rawalpindi, Pakistan)

Recent techniques of fuzzy set theory and its applications

Multi Criteria Decision Making (MCDM) provides strong decision making in domains where selection of best alternative is highly complex. In this article, will present the different techniques of fuzzy Multicriteria decision making named as soft sets, bipolar soft sets and their applications in different fields. Some new results of the said techniques and its connection in different paradigm will be presented.

Nikola MUTAVDŽIĆ

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Note on some classes of holomorphic functions related to Jack's and Schwarz's lemma

In this paper we discuss holomorphic mappings f of the unit disc $\mathbf{U}$ and corresponding index defined as $I_f(z) = zf'(z)/f(z)$. We are interested in finding bounds on the growth of functions f and related issues, if there are known some properties of I_f on $\mathbf{U}$. Our main tool in accomplishing this connection is Jack's lemma. As a special case, we got estimates on the growth of some classes of α-starlike functions as well as some interesting generalisations.

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SUNANDA NAIK

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A study of boundedness property of integral type operators on analytic function spaces

In this talk, we analyze the boundedness properties of certain operators on different analytic function spaces. We define an integral type operator $\mathcal{T}$,

$$\mathcal{T}f(z) = \int_0^1 f(tz)(1-tz)g(tz)\lambda(t)\mathrm{d}t,$$

where λ is a non-negative function, $g(z) = \sum_{n=0}^{\infty} P_n z^n$, where

$$\frac{1}{P_n} = \int_0^1 t^n \lambda(t)\mathrm{d}t$$

is a moment sequence and $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is analytic on the unit disc. In particular, if

$$P_n = \frac{\Gamma(c)\Gamma(b+1+n)}{\Gamma(b+1)\Gamma(c+n)},$$

we have a two parameter family of operators $\mathcal{P}^{b,c}$,

$$\mathcal{P}^{b,c}f(z) = \frac{1}{B(c,b+1-c)}\int_0^1 t^{c-1}(1-t)^{b-c}(1-tz)F(1,b+1;c;tz)f(tz)\mathrm{d}t$$

which is valid for $\mathrm{Re}(b+1) > \mathrm{Re}\, c > 0$ and $F(a,b;c;z)$ is the classical hypergeometric function. We call this a *generalized* Cesáro operator and discuss its boundedness property on various function spaces such as Hardy, BMOA and a-Bloch spaces. We find that $\mathcal{P}^{b,c}$ for $\mathrm{Re}(b+1) > \mathrm{Re}\, c > 0$ is bounded on H^p if and only if $p \in (0,\infty)$ and on B^a if and only if $a \in (1,\infty)$. It is unbounded on B^a, $a \in (0,1]$, H^∞ and BMOA. We also prove that if $f \in H^\infty$ then $\mathcal{P}^{b,c}f \in$ BMOA, with the condition $c > 2$. Also we supply an upper bound for norm of $\mathcal{P}^{b,c}$ from H^∞ to BMOA with this condition.

Key Words and Phrases: Gaussian hypergeometric functions, Cesáro operators.

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MARIA NOWAK

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Harmonically weighted Dirichlet space associated with finitely atomic measures and de Branges-Rovnyak spaces

We deal with the *de Branges-Rovnyak space* $\mathcal{H}(b)$ in the case when b is not an extreme point of the unit ball of H^∞. We describe the structure of such spaces $\mathcal{H}(b)$ for some functions b and their connections with the *harmonically weighted Dirichlet spaces* defined below.

For a finite measure μ on $\mathbf{T} = \partial\mathbf{D}$ let $P\mu$ denote the Poisson integral of μ given by

$$P_\mu(z) = \int_{\mathbf{T}} \frac{1-|z|^2}{|\zeta - z|^2}\mathrm{d}\mu(\zeta), \quad z \in \mathbf{D}.$$

The associated harmonically weighted Dirichlet space $\mathcal{D}(\mu)$ consists of functions f analytic in $\mathbf{D}$ for which

$$\mathcal{D}_\mu(f) = \int_{\mathbf{D}} |f'(z)|^2 P\mu(z)\mathrm{d}A(z) < \infty.$$

In 1997 D. Sarason showed that $\mathcal{D}(\delta_\lambda)$, where δ_λ is the unit mass at a point λ of $\mathbf{T}$ can be identified with $\mathcal{H}(b_\lambda)$, where $b_\lambda(z) = (1-w_0)\overline{\lambda}z/(1-w_0\overline{\lambda}z)$, and $w_0 = (3-\sqrt{5})/2$.

In our case we obtain equality $\mathcal{H}(b) = \mathcal{D}(\mu)$, where μ is a finite sum of atoms. We mention that for such measures μ some results on invariant subspaces of $\mathcal{D}(\mu)$ were obtained by D. Sarason in 1998.

Further results showing connection between the spaces $\mathcal{H}(b)$ and $D(\mu)$ have been recently obtained by T. Ransford, D. Guillot, N. Chevrot and C. Costara.

The talk is based on a joint work with Bartosz Łanucha.

Tania OSIPCHUK

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Some topological properties of generalized closed-to-convex sets

The properties of generalized closed-to-convex sets in the n-dimensional real Euclidean space $\mathbf{R}^n$, $n > 1$, also known as weakly m-semiconvex, $m = 1, 2, \ldots, n-1$, are investigated. An open set of $\mathbf{R}^n$ is called *weakly m-semiconvex* if, for any boundary point of the set, there exists an m-dimensional half-plane passing through this point and not intersecting the given set [1]. A closed set of $\mathbf{R}^n$ is called *weakly m-semiconvex* if it is approximated from the outside by a family of open weakly m-semiconvex sets [1]. A point of the complement of a set of $\mathbf{R}^n$ to the whole space is called an *m-nonsemiconvexity point* of the set if any m-dimensional half-plane passing through the point intersects the set.

Theorem 1. ([2], [4]) *Any open or closed weakly 1-semiconvex set of $\mathbf{R}^2$ having non-empty set of 1-nonsemiconvexity points consists of not less than three connected components.*

Theorem 2. ([3]) *Any open, bounded, and weakly 1-semiconvex set of $\mathbf{R}^2$ having smooth boundary and non-empty set of 1-nonsemiconvexity points consists of not less than four connected components.*

Theorem 3. ([4]) *Any closed, bounded, and weakly 1-semiconvex set of $\mathbf{R}^2$ having smooth boundary, non-empty interior, and non-empty set of 1-nonsemiconvexity points consists of not less than four connected components.*

Theorem 4. ([4]) *A non-empty interior of a closed, weakly 1-semiconvex set of $\mathbf{R}^2$ having a finite number of components is weakly 1-semiconvex.*

Theorem 5. ([4]) *There exist weakly m-semiconvex domains and closed connected sets of $\mathbf{R}^n$, $n \geq 3$, having non-empty set of m-nonsemiconvexity points, $1 \leq m < n-1$.*

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Rough set flow graph visualizer in classification and prediction software system (CLAPSS)

Classification and Prediction Software System (CLAPSS) [1] is a tool in which some specialized approaches based on fuzzy sets and rough sets are implemented (cf. [2], [3]). The tool is equipped with a user-friendly graphical interface. We present a new functionality, recently added to the CLAPSS, concerning visualizing rough set flow graphs defined by Z. Pawlak [5] as a tool for reasoning from data. On the basis of rough set flow graphs, it is also possible in CLAPSS to perform the temporal inference supported by fuzzy set operators (cf. [4]).

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On some generalization of Jackson q-derivative

For some complex ζ we introduce an operator $d_\zeta f(z)$ defined for the analytic functions f. For some values of ζ this operator becomes the Jackson q-derivative. We give some properties of this operator.

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SAMINATHAN PONNUSAMY

Indian Institute of Technology Madras (Chennai, India)

Landau-Bloch theorems for harmonic mappings

In this talk, we present properties of planar harmonic mappings. By using the subordination theory, a sharp coefficient estimate will be presented along with several applications. Results about Landau-Bloch's constant will be recalled: one for planar harmonic mappings and the other for $L(f)$, where L represents the linear complex operator $L = z\frac{\partial}{\partial z} - \overline{z}\frac{\partial}{\partial \overline{z}}$ defined on the class of complex-valued C^1 functions in the plane and f is an open harmonic mapping. Latest developments on Landau-Bloch's theorem for several different classes of harmonic and bi-analytic functions will be discussed.

Feliks PRZYTYCKI

Institute of Mathematics of Polish Academy of Sciences (Warsaw, Poland)

On Hausdorff dimension of Julia sets

I will discuss Hausdorff (fractal) dimension of limit Julia sets arising when iterating complex polynomials or rational functions on the Riemann sphere. Methods used, belong to the thermodynamic formalism involving Kolmogorov entropy, Lyapunov exponents and geometric pressure and equilibria. A relation with the boundary behaviour of conformal maps on the unit disc will be mentioned.

Jouni RÄTTYÄ

University of Eastern Finland (Joensuu, Finland)

Bergman projection and BMO in hyperbolic metric

The Bergman projection P_α, induced by a standard radial weight, is bounded and onto from L^∞ to the Bloch space $\mathcal{B}$. However, $P_\alpha : L^\infty \to \mathcal{B}$ is not a projection. This fact can be emended via the boundedness of the operator $P_\alpha : \mathrm{BMO}_2(\Delta) \to \mathcal{B}$, where $\mathrm{BMO}_2(\Delta)$ is the space of functions of bounded mean oscillation in the Bergman metric.

We consider the Bergman projection P_ω and the space $\mathrm{BMO}_{\omega,p}(\Delta)$ of functions of bounded mean oscillation induced by $1 < p < \infty$ and a radial weight $\omega \in \mathcal{M}$. Here $\mathcal{M}$ is a wide class of radial weights defined by means of moments of the weight, and it contains the standard and the exponential-type weights. We describe the weights such that $P_\omega : \mathrm{BMO}_{\omega,p}(\Delta) \to \mathcal{B}$ is bounded. They coincide with the weights for which $P_\omega : L^\infty \to \mathcal{B}$ is bounded and onto. This result seems to be new even for the standard radial weights when $p \neq 2$.

V. RAVICHANDRAN

National Institute of Technology (Tiruchirappalli, India)

Radius problems in Geometric Function Theory

It is well-known that every starlike univalent function on the unit disk $\mathbf{D}$ need not be convex in $\mathbf{D}$. The Koebe function $k : \mathbf{D} \to \mathbf{C}$ defined by $k(z) = z/(1-z)^2$ is starlike and it maps the disk $\mathbf{D}_\rho$ onto a convex domain for any $0 < \rho \leqslant 2 - \sqrt{3}$ but not for any larger value. The same is the true for any starlike function and the number $2-\sqrt{3}$ is known as the radius of convexity of starlike functions. Given a property P and a class $\mathcal{M}$, the radius of the property P in the class $\mathcal{M}$ is the largest number R such that every function in the class $\mathcal{M}$ has the property in the disk $\mathbf{D}_r$ for every $r < R$ and for some function the property P does not hold in any larger disk. The radius problems for subclasses of univalent functions can be solved in many ways. We shall discuss some of these techniques in this talk, in particular, the use standard estimates for function with positive real part, closure property of classes under convolution with convex functions, subordination theory as well as the extreme point theory.

Mohsan RAZA

Government College University Faisalabad (Faisalabad, Pakistan)

Starlikeness associated with the reciprocal of Bernoulli functions

Let

$$\varphi_{\mathfrak{B}}(z) := \frac{z}{\mathrm{e}^z - 1} = 1 - \frac{z}{2} + \sum_{n=1}^{\infty} \frac{B_{2n}}{(2n)!} z^{2n}, \quad z \in \mathbf{D} := \{z \in \mathbf{C} : |z| < 1,$$

where the constants B_{2n} are Bernoulli numbers. Let $\mathcal{S}^*_{\mathfrak{B}}$ denote the class of normalized analytic functions f such that

$$\frac{zf'(z)}{f(z)} \prec \frac{e^z - 1}{z}, \quad z \in \mathbf{D}.$$

For this class, we obtain a structural formula, inclusion results, differential subordinations and some sharp radii constants such as radius of convexity, radius for the class of Janowski starlike functions and radius for some other subclasses of starlike functions. Moreover, we get sharp results for initial coefficients for the functions in this class. Furthermore, we obtain sharp bounds of the Hankel determinants of order two and three.

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Swadesh Kumar SAHOO

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Geometry of Cassini ovals and applications

The presentation would be as simple as possible for audience. The talk mainly includes the basic definition of Cassini ovals with some applications to Earth's orbit and population growth. Theoretically, we consider a distance function that involves the Cassini ovals and present some of its geometric properties using basic complex analysis and elementary geometry.

David SHOIKHET

The Galilee Research Center for Applied Mathematics, ORT College Braude (Karmiel, Israel)

and

Holon Institute of Technology (Holon, Israel)

Holomorphic mappings and semigroups, rigidity and fixed points

There is a long history associated with the problem of iterating nonexpansive and holomorphic mappings and finding their fixed points.

Historically, complex dynamics and geometrical function theory have been intensively developed from the beginning of the twentieth century. They provide the foundations for broad areas of mathematics. In the last fifty years the theory of holomorphic mappings on complex spaces has been studied by many mathematicians with many applications to nonlinear analysis, functional analysis, differential equations, classical and quantum mechanics. The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system:

$$\frac{dx}{dt} + f(x) = 0,$$

where x is a variable describing the state of the system under study, and f is a vector-function of x. The study of such systems when f is a monotone or an accretive (generally nonlinear) operator on the underlying space has recently been the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems.

In this talk we give a brief description of the classical statements with their modern interpretations for discrete and continuous semigroups of holomorphic and hyperbolically nonexpansive mappings in Hilbert and Banach spaces. We also present some special recent achievements for the one-dimensional case.

Young Jae SIM[1], Derek K. THOMAS[2]

[1]*Kyungsung University* (Busan, Korea)
[2]*Swansea University Bay Campus* (Swansea, United Kingdom)

Recent bounds for Hankel determinants for starlike functions with respect to symmetrical points

We present a survey of some recent significant advances in finding sharp bounds for Hankel determinants for functions which are starlike with respect to symmetrical points.

Sanjeev SINGH

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The generalized Marcum function of the second kind: Monotonicity patterns and tight bounds

We present a detailed analysis (monotonicity, convexity, recurrence relation, closed-form expression and tight bounds) of a new special function which we call the generalized Marcum function of the second kind, which is an analogous survival (or reliability) function to the so-called generalized Marcum Q-function or the generalized Marcum function of the first kind (the survival function of the non-central chi distribution). The main difference between these two generalized Marcum functions is that they involve the modified Bessel functions of the first and second kinds.

Srikandan SIVASUBRAMANIAN

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On a subclass of close-to-convex functions involving nephroid and cardioid domains

Let $\mathscr{H}$ be the class of all holomorphic functions in the open unit disc $\mathbf{D} := \{z \in \mathbf{C} : \ |z| < 1\}$. Further, let $\mathscr{A}$ characterize the subclass of $\mathscr{H}$ entailing of functions h with the normalization $h(0) = h'(0) - 1 = 0$. Hence, the class of all functions $h \in \mathscr{A}$ will be of the appearance

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbf{D}.$$

By $\mathscr{S}$, we indicate the subclass of $\mathscr{A}$ consisting of univalent functions. A function $f \in \mathscr{H}$ is subordinate to a new function $g \in \mathscr{H}$ written as $f \prec g$ if there exists a function $\omega \in \mathscr{H}$ satisfying $\omega(0) = 0$, $\omega(\mathbf{D}) \subset \mathbf{D}$ and such that $f(z) = g(\omega(z))$ for every $z \in \mathbf{D}$. In specific, if g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathbf{D}) \subset g(\mathbf{D})$.

In the present talk, we initiate and look into a new class of holomorphic functions in the open unit disc with a adapted version of the known Robertson's analytic formula for starlike functions with respect to a boundary point by subordinating with a function that are coupled with a nephroid and a cardioid domain. For the newly defined classes, apart from the initial coefficients and the recognizable coefficient functional, the authors also attain certain differential subordination results connecting the class of functions relating nephroid domain and cardioid domain. The connections of the main results obtained in this article are also compared with the earlier results existing in the literature.

This is a joint work with Adam Lecko and Gangadharan Murugusundaramoorthy.

AMS Subject Classification: Primary 30C45, 30C50; Secondary 30C80

Key Words and Phrases: holomorphic, univalent, starlike function of order β, starlike function with respect to a boundary point, subordination, coefficient estimates.

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Janusz SOKÓŁ

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On some new length problem for analytic functions

Let $\mathcal{H}$ denote the class of analytic functions in the unit disk $|z| < 1$. Let $C(r, f)$ be the closed curve which is the image of the circle $|z| = r < 1$ under the mapping $w = f(z) \in \mathcal{H}$, $L(r, f)$ the length of $C(r, f)$ and let $A(r, f)$ be the area enclosed by $C(r, f)$. Let $l(re^{i\theta}, f)$ be the length of the image curve of the line segment joining $re^{i\theta}$ and $re^{i(\theta+\pi)}$ under the mapping $w = f(z)$ and let $l(r, f) = \max_{0\leq\theta<2\pi} l(re^{i\theta}, f)$. We find upper bound for $l(r, f)$ for $f(z)$ in some known classes of functions. Moreover, we prove that $l(r, f) = \mathcal{O}\left(\log \frac{1}{1-r}\right)$ and that $L(r, f) = \mathcal{O}\left\{A(r, f) \log \frac{1}{1-r}\right\}^{1/2}$ as $r \to 1$ under weaker assumptions on $f(z)$ than some previous results of this type.

Bożena STARUCH

University of Warmia and Mazury in Olsztyn (Olsztyn, Poland)

On the problem of effective use of fabric remnants in the production of upholstered furniture

We consider the problem of the use of remnants of upholstery fabrics. This significant problem arises in the management of upholstered furniture factories with demand-driven manufacturing. As upholstery fabrics cost is one of the main factors influencing production costs, there is a need to use the remnants of fabrics efficiently. This problem looks very similar to the following NP-hard problems: a bin packing problem, a 1-dimensional cutting stock problem, a variable sized bin packing problem, a multi-knapsack problem. However, it is much more complicated, because the main feature of demand-driven manufacturing is a huge variability of orders and thus a large variability of product assortment. Moreover, the objective is ambiguous. In this paper, we propose a method of modeling the objective function and give an algorithmic solution that meets practitioners requirements.

Toshiyuki SUGAWA

Tohoku University (Tohoku, Japan)

Harmonic spirallike functions and harmonic strongly starlike functions

In this talk, we introduce two classes of harmonic univalent functions of the unit disk, called hereditarily λ-spirallike functions and hereditarily strongly starlike functions, which are analogues of λ-spirallike functions and strongly starlike functions, respectively. We note that a relation can be obtained between these two classes. We also investigate analytic characterization of hereditarily spirallike functions and uniform boundedness of hereditarily strongly starlike functions. Some coefficient conditions are also given for hereditary strong starlikeness and hereditary spirallikeness. As a simple example, we consider harmonic functions of a special form.

This talk is based on the joint paper with Xiu-Shuang Ma (China) and S. Ponnusamy (India), which appears in Monatsh. Math. (https://doi.org/10.1007/s00605-022-01708-y).

Zbigniew SURAJ

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On some approach to approximate real-time decision making: theory and implementation

One of the important and current research problems in computer science and new artificial intelligence [4] is building real-time systems that can operate reliably in various environments, including uncertain and imprecise environments [3],[16]. A good theoretical basis for research in this area may be the theories of rough sets [7],[6],[5], fuzzy sets [15],[8] and Petri nets [9],[2], especially fuzzy Petri nets [3],[16],[1],[11].

The aim of the work is to present both an approach to the construction of a concurrent algorithm represented by a fuzzy Petri net, and its practical implementation in PNeS (Petri Net System) [10]. In our approach, the concurrent algorithm is constructed on the basis of knowledge obtained from empirical data stored in a given decision table [7] representing the fragment of reality we are interested in. In addition, we will show how easy it is to calculate the values of the parameters characterizing the decision rules obtained from the data table, which are the basis for building a concurrent algorithm. Considering the literature on the subject, it can be seen that it is usually devoted to the description of a similar problem, but using the knowledge of field experts and not empirical data [16],[3].

It should also be noted that the net model presented here allows for the fastest possible identification of objects specified in a given decision table. This effect is achieved thanks to the use of both true and acceptable rules in the construction of a concurrent algorithm, as well as the appropriate organization of its work. Additionally, the presented approach uses an original interpretation of the net arc weights, based on the concepts of rough set theory [5]. As a result, arc weight values can be easily and directly determined from a given decision table [13],[12] and not by a field expert [3],[16],[1]. The proposed approach also allows the use of any triangular norms to describe the behavior of the WPFP-net [12]. This in turn makes the resulting nets even more suitable for describing real situations. In order to show potential usefulness in modeling decision making in an uncertain environment, as well as to check the practical effectiveness of the proposed model, we implemented this methodology in PNeS [14]. By using this new functionality in PNeS, quite advanced experimental research can be carried out related to the automated construction of concurrent algorithms based on decision tables describing both the structure and behavior of modeled real-time systems.

In many situations of everyday life, we are not able to determine the exact value of the degree of belonging of a given element to the set under consideration or the exact value of the degree of truthfulness of a logical sentence, so in further research we should focus on the interval fuzzy sets [8] rather than on classical fuzzy sets [15] to indicate a range of desired values instead of their exact values. Therefore, in modeling the concurrent algorithm, WPFP-nets that use triangular interval norms instead of the classical triangular norms can be useful. Such a modification of the approach should make the proposed net model even more flexible, general and therefore practical. This is one example of a problem that we would like to investigate using the approach presented in this work.

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The Schwarz lemma for HQR mappings

In this talk, we consider harmonic quasiregular mappings from the unit disk $\mathbf{U} = \{z \in \mathbf{C} : |z| < 1\}$ to itself, to the vertical strip $\mathbf{S} = \{z \in \mathbf{C} : -1 < \operatorname{Re} z < 1\}$ and to the right half-plane $\mathbf{K} = \{z \in \mathbf{C} : \operatorname{Re} z > 0\}$. For such mappings we formulate and prove the Schwarz lemma and the Schwarz-Pick lemma. These results can be viewed as generalizations and analogies of the corresponding theorems for holomorphic mappings. Note that the hyperbolic metric on the planar domains plays a crucial role in our investigations.

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Geometric properties of ratios of hypergeometric functions

Gaussian hypergeometric functions play vital role in the geometric function theory. One of the important influence of hypergeometric functions is in obtaining the order of starlikenees. This was highly motivated by the work of Kustner, by using the continued fraction expansion for the ratio of two Gaussian hypergeometric functions. The same continued fraction can be used to study another concept called characterization of Pick functions, which is of equal importance in geometric function theory. Besides providing the underlying literature in the above directions, generalization of the ratio of hypergeometric functions to another class of special functions (generalized polylogarithm) will be given in this talk. New results and open problem in this direction will also be provided.

Gabriela SZAJNOWSKA

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Properties of the Green's function for a third-order boundary value problem

We will deal with the following third order differential equation

$$-y''' + m^2 y' = 0, \tag{1}$$

where m is a positive parameter, coupled with the two point boundary value conditions

$$y(0) = 0, \quad y'(0) = 0, \quad y'(1) = 0. \tag{2}$$

It is the purpose of the poster to present the properties of the Green's function for the above problem that may be used to obtain sufficient conditions implying the existence of positive solutions to nonhomogeneous problem corresponding to (1)-(2). The results of the poster are based on [1]-[3].

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Assessment of learning outcomes after the first semester of studies in mathematics conducted by the University of Rzeszów in the years 2019-2022. Comparison.

We will present the results of the study on the impact of educational forms on the achievement of learning outcomes. The analysis will concern the degree of obtaining learning outcomes pursued in mathematical subjects in the first semester of study. Its relationship with the results of the matura exams will also be presented. The data will be compared for three groups of students starting their studies in the field of mathematics in the academic years 2019/2020, 2020/2021 and 2021/2022 respectively.

All the groups had the same recruitment rules and the study programme. The conditions of preparation for the matura exam and the forms of education in the first semester (full-time, remote and hybrid education) were the variables in the study.

The results of this study are designed to help lecturers to bring the problem closer and make the necessary changes in teaching and assessment methods.

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Coefficient functionals for alpha-convex functions associated with the exponential function

Let $\mathcal{H}$ denote the class of all analytic functions in $\mathbf{D} := \{z \in \mathbf{C} : |z| < 1\}$ and $\mathcal{A}$ be the subclass of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathbf{D}.$$

Denote by $\mathcal{S} \subset \mathcal{A}$ the subclass of univalent functions.

For $\alpha \in [0,1]$, denote by $\mathcal{M}_\alpha \subset \mathcal{A}$, the so-called α-convex functions f satisfying

$$\operatorname{Re}\left\{(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right)\right\} > 0, \quad z \in \mathbf{D}.$$

We say that a function $f \in \mathcal{H}$ is subordinate to a function $g \in \mathcal{H}$, if there exists a function $\omega \in \mathcal{H}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ for $z \in \mathbf{D}$ (called a Schwarz function), such that $f(z) = g(\omega(z))$ for $z \in \mathbf{D}$. We write $f \prec g$. If g is univalent and $f(0) = g(0)$, then $f \prec g$ is equivalent to $f(\mathbf{D}) \subseteq g(\mathbf{D})$.

Definition. *A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{M}_\alpha(\exp)$, $\alpha \in [0,1]$, if f satisfies the following condition:*

$$(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \exp(z), \quad z \in \mathbf{D}.$$

In the speech, we find the sharp bound for $|H_{2,2}(f)|$ when $f \in \mathcal{M}_\alpha(\exp)$, $\alpha \in [0,1]$, together with the sharp bound of the functional

$$|J_{2,3}(f)| := |a_2 a_3 - a_4|,$$

when $f \in \mathcal{M}_\alpha(\exp)$, $\alpha \in [0,1]$.

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Świątkowski-type conditions

By $I(a,b)$ we denote the open interval with ends a, b. $C(f)$ stands for the set of all continuity points of a function $f\colon \mathbf{R} \to \mathbf{R}$.

In 1978 Mańk and Świątkowski introduced the notion of a new property of functions, now called the Świątkowski condition ([3]):

Definition 1. *We say that f satisfies the Świątkowski condition (or is a Świątkowski function) if for all $a \neq b$ with $f(a) < f(b)$ there is a point $x \in I(a,b) \cap C(f)$ such that $f(a) < f(x) < f(b)$.*

In [2] Maliszewski gave the stronger form of the above condition:

Definition 2. *We say that f satisfies strong Świątkowski condition (or f is a strong Świątkowski function) if for all $a \neq b$ and each y between $f(a)$ and $f(b)$, there is an $x \in I(a,b) \cap C(f)$ for which $f(x) = y$.*

Recently in the paper [1] the authors relieved Definition 1 from demanding that f has to be continuous at x:

Definition 3. *We say that f satisfies the weak Świątkowski condition (or is a weakly Świątkowski function) if for all $a \neq b$ with $f(a) < f(b)$ there is a point $x \in I(a, b)$ such that $f(a) < f(x) < f(b)$.*

In the talk we will present the properties of families of functions fulfilling the above conditions. We will compare them with some other families of functions (cliquish, Darboux) and examine their algebraic properties (among them lineability and algebrability).

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Coefficient invariances for convex functions

We present a survey of known invariances between coefficient functionals for convex univalent functions and the corresponding coefficient functionals for the inverse function, highlighting some curious results and suggesting problems for further research.

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On difference of successive coefficients in $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$

Let $\mathcal{A}$ be the family of all functions analytic in the open unit disk $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$ having the power series expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

In this paper we consider coefficient problems in the following classes:

$$\mathcal{M}(\alpha) = \left\{ f \in \mathcal{A} : \mathfrak{Re} \left\{ \frac{zf'(z)}{f(z)} \right\} < 1 + \frac{\alpha}{2} \right\}, \quad \alpha > 0,$$

$$\mathcal{N}(\alpha) = \left\{ f \in \mathcal{A} : \mathfrak{Re} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} < 1 + \frac{\alpha}{2} \right\}, \quad \alpha > 0.$$

In the first part, we find the coefficient bounds for the moduli of the Taylor coefficients a_n of functions from $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$.

In the second part, we investigate the differences of the moduli of successive coefficients $|a_{n+1}| - |a_n|$ of functions from $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$. We obtain upper and lower estimates of these differences.

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Estimates of coefficient functionals for close-to-convex functions

Let $\mathcal{A}$ be the class of functions analytic in the open unit disc $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$ and normalized by the condition $f(0) = 0 = f'(0) - 1$ and let $\mathcal{S}^*$ be its subclass consisting of starlike functions. We say that a function $f \in \mathcal{A}$ is *close-to-convex with argument* β *with respect to* g if there exists a real parameter $\beta \in (-\pi/2, \pi/2)$ and a function $g \in \mathcal{S}^*$ such that

$$\operatorname{Re}\left\{\frac{e^{i\beta} z f'(z)}{g(z)}\right\} > 0 \tag{1}$$

for all $z \in \mathbf{D}$. The family of these functions will be denoted by $\mathcal{C}_\beta(g)$. It is obvious that

$$\bigcup_{\beta\in(-\pi/2,\pi/2)} \bigcup_{g\in\mathcal{S}^*} \mathcal{C}_\beta(g) = \mathcal{C}$$

where $\mathcal{C}$ denotes the well known class of all close-to-convex functions (see [1], [2]). Further for a given $-1 \le \alpha \le 1$ we define a function

$$F_\alpha(z) = \frac{z}{1-\alpha z^2} = z + \sum_{n=2}^{\infty} \alpha^n z^{2n+1}, \quad z \in \mathbf{D},$$

starlike for all $\alpha \in [-1, 1]$. Let us consider the class $\mathcal{C}_0(F_\alpha)$. Then (1) takes a form

$$\operatorname{Re}\{(1-\alpha z^2) f'(z)\} > 0, \quad z \in \mathbf{D}.$$

It is worth noting that $\mathcal{C}_0(F_\alpha)$ is the subclass of the family $\mathcal{C}(\beta, \xi_1, \xi_2)$ of analytic functions defined by the formula

$$\operatorname{Re}\left\{e^{i\beta}(1-\xi_1 z)(1-\xi_2 z) f'(z)\right\} \ge 0, \quad z \in \mathbf{D},$$

with $\beta \in (-\pi/2, \pi/2)$ and complex parameters $\xi_1, \xi_2 \in \overline{\mathbf{D}} = \{z \in \mathbf{C} : |z| \le 1\}$. This class was introduced by A. Lecko in 1994 [3] and has been extensively examined (see [4]-[6]). Furthermore, if $0 \le \alpha \le 1$, then $\mathcal{C}_0(F_\alpha) \equiv \mathcal{C}_0(\sqrt{\alpha})$, where the symbol $\mathcal{C}_\beta(\alpha)$ ($\alpha \in [0,1]$, $\beta \in (-\pi/2, \pi/2)$) stands for the class of analytic functions that satisfy the condition

$$\operatorname{Re}\left\{e^{i\beta}(1-\alpha^2 z^2) f'(z)\right\} > 0$$

in $\mathbf{D}$. The family $\mathcal{C}_\beta(\alpha)$ was defined in 1989 [7] (see also [8]).

Recently, there have been many articles on univalent functions with a fixed second coefficient in Taylor series expansion (see for example [9], [10]). Motivated by this idea we will focus on the problem of finding estimates of some coefficient functionals for $f \in \mathcal{C}_0(F_\alpha)$ with the second coefficient fixed.

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Some properties of functions from family of even holomorphic functions of $\mathbf{C}^n$

Let us consider some properties for a family of even holomorphic functions defined on bounded complete n-circular domain G of $\mathbf{C}^n$ (see [5]). In particular, we will present relationships between this family and some of type of Bavrin's families (see [1]) and the estimates of G-balances of homogeneous polynomials. Furthemore, we will give a sufficient condition for functions belonging to the family under consideration.

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O pewnym podstawowym problemie w początkowym etapie nauczania analizy matematycznej

W trakcie referatu chciałbym zwrócić uwagę na problemy dydaktyczne związane z wprowadzeniem zbioru liczb rzeczywistych w ramach wykładów z analizy matematycznej w odniesieniu do wiedzy absolwenta szkoły ponadpodstawowej oraz przedstawić tego konsekwencje dla omówienia podstawowych twierdzeń egzystencjalnych. Powyższe będzie oparte o analizę różnych podejść zaprezentowanych w publikacjach [1], [2], [3], [4].

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Applications of hypergeometric functions in the theory of differential subordinations

In this talk, we will discuss some applications of hypergeometric functions in solving certain kind of subordination implication problems. In particular, we use the geometrical properties of the Gaussian and the confluent hypergeometric functions to determine the best possible values of the real β so that for some analytic function p defined in the open unit disk $\mathbf{D}$ satisfying $p(0) = 1$, the differential subordination implication

$$p(z) + \beta z p'(z) \prec \mathcal{P}(z) \implies p(z) \prec 1 + \alpha z e^{z}, \quad z \in \mathbf{D},\ 0 < \alpha \leq 1,$$

holds, where $\mathcal{P}$ is any one of the following Carathéodory functions:

(1) $\sqrt{1+z}$,
(2) $1+z$,
(3) e^z.

We note that for each $\alpha \in (0,1]$, the function $\mathbf{D} \ni z \mapsto \varphi_c(z,\alpha) := 1 + \alpha z e^z$ maps $\mathbf{D}$ univalently onto the interior of a cardioid which is starlike with respect to $\varphi_c(0,\alpha) = 1$ and symmetric about the real-line.

As applications, we establish conditions which sufficiently ensure that a normalized analytic function f is a member of the Ma-Minda family $\mathcal{S}_c^*(\alpha)$ characterized by

$$f \in \mathcal{S}_c^*(\alpha) \iff \frac{zf'(z)}{f(z)} \prec 1 + \alpha z e^z, \qquad z \in \mathbf{D}.$$

Key Words and Phrases: hypergeometric functions, differential subordinations, starlike functions, cardioid, Bernoulli's lemniscate.

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A fixed point theorem in $B(H, \ell_\infty)$

In 1955, motivated by work of Dixmier and Day on uniformly bounded group representations, Kadison [3] raised the problem of whether every bounded homomorphism u from C^*-algebra $\mathcal{A}$ into $B(H)$ is similar to a *-homomorphism, i.e., does there exist an invertible operator $S \in B(H)$ such that the map $\tilde{u}(x) = S^{-1}u(x)S$ satisfies $\tilde{u}(x^*) = \tilde{u}(x)^*$? This problem, perhaps comparable with the Kadison–Singer problem recently solved in [5], is still open although important partial results were obtained by Christensen, Haagerup and others (see [1, 2]). It is also equivalent to a number of other important problems:

- is every bounded homomorphism $u : \mathcal{A} \to B(H)$ completely bounded? ([2])
- the derivation problem [4]–given a C^*-subalgebra $\mathcal{A} \subset B(H)$, is every derivation $\delta : \mathcal{A} \to B(H)$ inner?
- do all C^*-algebras have finite length? ([6])
- Dixmier's invariant operator range problem from 1950 ([7])
- are all von Neumann algebras hyperreflexive? (see [7]).

The complete list of equivalent formulations is longer.

The aim of this talk is to present a new approach to the Kadison similarity problem based on a fixed point theorem for isometries in $B(H, \ell_\infty)$.

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Technical applications of some results on plane harmonic mappings

Within this lecture the author will present some results on boundary normalized harmonic mappings of the unit disc onto (into) itself. As a direct application of the results we will present some technical design being its consequences.

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